



CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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9231/21

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

- 1 Find the Maclaurin's series for $e^x \tan x$ from first principles up to and including the term in x^2 . [5]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.

2 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Use the characteristic equation of $\mathbf{A}$ to show that

$$\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I},$$

where p and q are integers to be determined.

[6]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

3 The curve C has equation

$$xy^3 - 4x^3y = 3.$$

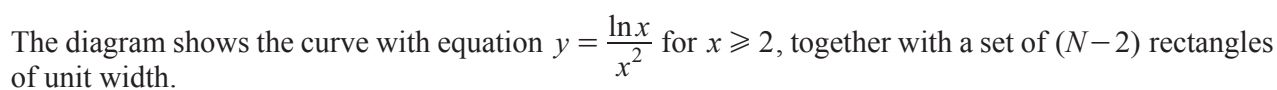
(a) Show that, at the point $(-1, 1)$ on C , $\frac{dy}{dx} = 11$. [3]

[illegible]

(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(-1, 1)$.

[5]

[illegible]


$$\sum_{r=1}^N \frac{\ln r}{r^2} < \frac{2+3 \ln 2}{4} - \frac{1+\ln N}{N}. \quad [7]$$
This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Use a similar method to find, in terms of N , a lower bound for $\sum_{r=1}^N \frac{\ln r}{r^2}$. [3]

[illegible]

5 Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\cos x,$$

given that, when $x = 0$, $y = -4$ and $\frac{dy}{dx} = 3$. [11]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

6 (a) Use de Moivre's theorem to show that

$$\operatorname{cosec} 5\theta = \frac{\operatorname{cosec}^5 \theta}{5 \operatorname{cosec}^4 \theta - 20 \operatorname{cosec}^2 \theta + 16}. \quad [6]$$

[illegible]

(b) Hence obtain the roots of the equation

$$x^5 - 10x^4 + 40x^2 - 32 = 0$$

in the form $\operatorname{cosec}(q\pi)$, where q is rational.

[4]

[illegible]

7 (a) Show that an appropriate integrating factor for

$$\sqrt{x^2-1}\frac{dy}{dx}+y=x^2-x\sqrt{x^2-1}$$

is $x + \sqrt{x^2 - 1}$.

[4]

[illegible]

(b) Hence find the solution of the differential equation

$$\sqrt{x^2-1}\frac{dy}{dx}+y=x^2-x\sqrt{x^2-1}$$

for which $y = 1$ when $x = \frac{5}{4}$. Give your answer in the form $y = f(x)$. [7]

This image shows a full page of a worksheet designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, and there are no margins, text, or other markings present.

- 8 (a)** Starting from the definition of $\cosh$ in terms of exponentials, prove that

$$2 \cosh^2 A = \cosh 2A + 1. \quad [3]$$

The curve C has parametric equations

$$x = 2 \cosh 2t + 3t, \quad y = \frac{3}{2} \cosh 2t - 4t, \quad \text{for } -\frac{1}{2} \leq t \leq \frac{1}{2}.$$

The area of the surface generated when C is rotated through 2π radians about the y -axis is denoted by A .

(b) (i) Show that $A = 10\pi \int_{-\frac{1}{2}}^{\frac{1}{2}} (2 \cosh 2t + 3t) \cosh 2t \, dt$. [4]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

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